MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

08-04-2019 Online (Evening)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above..

PART-A : CHEMISTRY

1. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are:

(1) 4.9 and 0 (2) 0 and 5.92 (3) 2.84 and 5.92 (4*) 0 and 4.9

Sol $[Fe(CN)_6]^{4-}$ $[Fe(H_2O)_6]^{2+}$

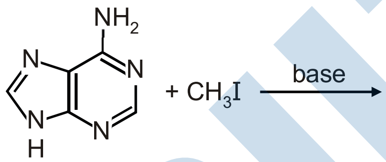
$Fe^{2+} \longrightarrow 3d^6$ $Fe^{2+} \longrightarrow 3d^6$

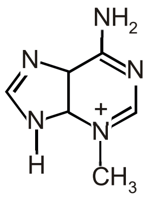
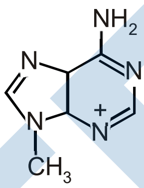
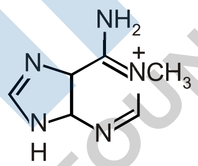
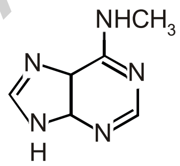
$t^6_{2g}eg^0$ $t^4_{2g}\ eg^2$

$\mu = \sqrt{n(n+2)}$ $\mu = \sqrt{n(n+2)} = \sqrt{24} = 4.9BM$

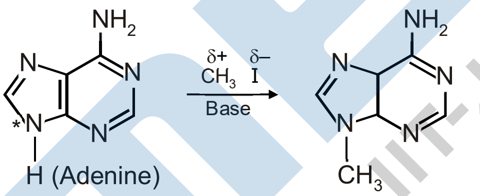
$\mu = 0$

2. The major product in the following reaction is:



(1)  (2*)  (3)  (4) 

Ans. **BONUS**

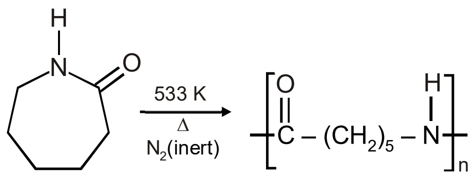
Sol. 

The nitrogen atom marked with * contain high electron density. It is attached to one Hatom which can be removed by base OH^- and the CH_3 group gets attached there.

3 The covalent alkaline earth metal halide (X = Cl, Br, I) is

(1) MgX_2 (2*) BeX_2 (3) SrX_2 (4) CaX_2

Sol. The direct monomer of Nylon-6 is caprolactum which polymerises to give Nylon-6 as follows:



Nylon-6

4. Calculate the standard cell potential (in V) of the cell in which following reaction takes place:

Fe^{2+} (aq) + Ag^{+} (aq) $\rightarrow$ Fe^{3+} (aq) + Ag(s)

Given that

$E^0_{Ag^+/Ag} = xV$

$E^0_{Fe^{2+}/Fe} = yV$

$E^0_{Fe^{3+}/Fe} = zV$

(1) x – y (2) x – z (3*) x + 2y – 3z (4) x + y – z

Sol. Given:

$E^0_{Ag'/Ag} = x$ - - - - - - (1)

$E^0_{Fe^{2+}/Fe} = y$ - - - - - (2)

$E^0_{Fe^{3+}/Fe} = z$ - - - - - (3)

Using equation:

$\Delta G^0 = -nFE^0$

$\Delta G^0_1 = -Fx$

$\Delta G^0_2 = -2Fy$

$\Delta G^0_3 = -3Fz$

$Fe^{2+} + 2e^- \longrightarrow Fe - 2Fy$

$Fe^{3+} + 3e^- \longrightarrow Fe - 3Fz$

$Fe^{2+} \longrightarrow Fe^{3+} + e^-(-2Fy + 3Fz)$

$Ag^{2+} + e^- \longrightarrow Ag - Fx$

$\Delta G_{Total} = -2Fy + 3Fz + 3Fz - Fx = -FE^0_{cell}$

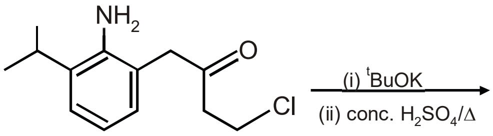
$E^0_{cell} = x + 2y - 3z$

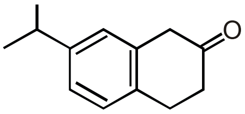
5. The IUP AC symbol for the element with atomic number 119 would be :

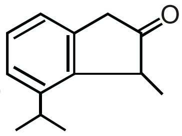
(1*) uue (2) une (3) uun (4) unh

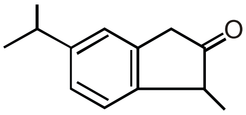
Sol. The IUPAC name of element having atomic number 119 is "Ununennium". So its symbol is uue.

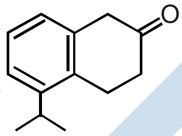
6. The major product of the following reaction is:



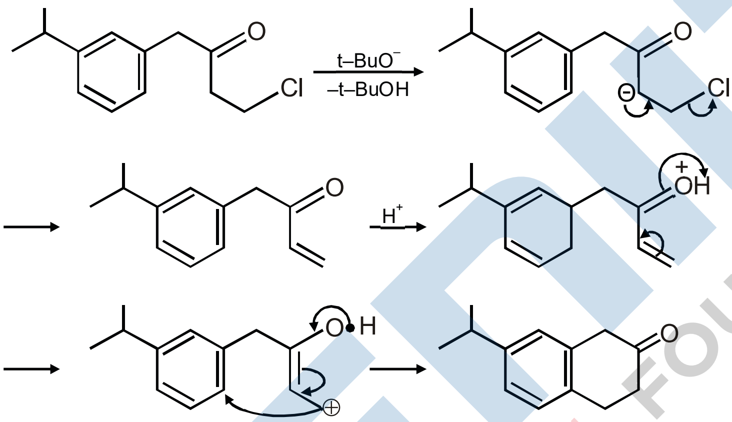
(1*) 

(2) 

(3) 

(4) 

Sol.

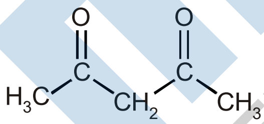


7. Which of the following compounds will show the maximum 'enol' content?

(1) $CH_3COCH_2CONH_2$ (2) $CH_3COCH_2COOC_2H_5$

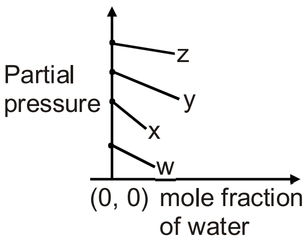
(3*) $CH_3COCH_2COCH_3$ (4) CH_3COCH_3

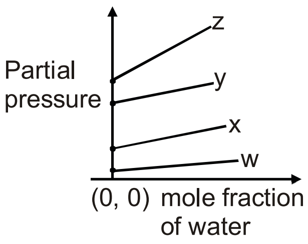
Sol.

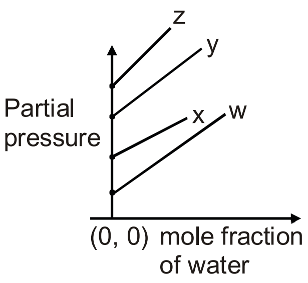


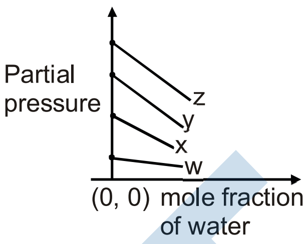
Due to presence of active methylene group and stabilization of enol by intramoelcular H bond forming 6 membered ring structure.

8. For the solution of the gases w, x, y, and z in water at 298 K, the Henry's law constants (K_H) are 0.5, 2, 35 and 40 k bar, respectively. The correct plot for the given data is:

(1) 

(2) 

(3) 

(4*) 

Sol. From Henry's law

$P_{gas} = K_H.X_{gas}$

$P_{gas} = K_H(1 - X_{H_2O})$

$P_{gas} = K_H - K_H X_{H_2O}$

Comparing this equation with straight line equation y = mx + c

9. The correct statement about ICl_5 and ICl_4^- is :

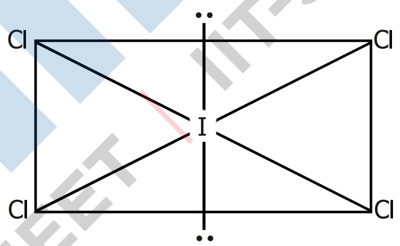
(1) both are isostructural

(2) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral.

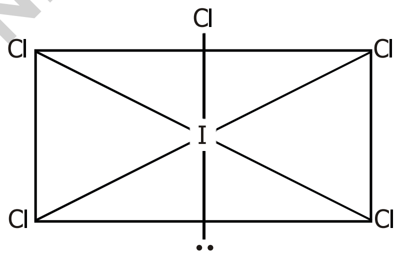
(3) ICl_5 is square pyramidal and ICl_4^- is tetrahedral.

(4*) ICl_5 is square pyramidal and ICl_4^- is square planar

Sol. $[ICl_4]^- \rightarrow sp^3d^2$



$[ICl_5] \rightarrow sp^3d^2$



10. Polysubstitution is a major drawback in

(1) Reimer Tiemann reaction (2) Friedel Craft's acylation

(3*) Friedel Craft's alkylation (4) Acetylation of aniline

Sol. Polysubstitution is a major drawback of Friedal craft's alkylation. It is so because activating behaviour of benzene ring increases with increase in number of alkyl group on benzene ring.

11. The statement that is INCORRECT about the interstitial compounds is :

(1*) They are chemically reactive (2) They have metallic conductivity

(3) They are very hard (4) They have high melting points

Sol. In case of interstial compounds there is presence of small atoms (or impurity) in the lattice of metal. So interstial compounds are hard, high melting point and chemically inert.

12. The maximum prescribed concentration of copper in drinking water is :

(1) 0.05 ppm (2*) 3 ppm (3) 0.5 ppm (4) 5 ppm

Sol. Maximum prescribed concentration of copper in drinking water is 3 ppm. Above this concentration water becomes toxic.

13. Among the following molecules/ions.C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2

Which one is diamagnetic and has the shortest bond length?

(1*) C_2^{2-} (2) O_2 (3) O_2^{2-} (4) N_2^{2-}

14. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ then for 1.5 p momentum of the photoelectron, the wavelength of the light should be: (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function) :

(1*) $\frac{4}{9}\lambda$ (2) $\frac{1}{2}\lambda$ (3) $\frac{3}{4}\lambda$ (4) $\frac{2}{3}\lambda$

Sol. $K.E. = \frac{1}{2}mu^2 = \frac{1}{2}\frac{m^2u^2}{m} = \frac{1}{2}\frac{P^2}{m}$

$$\frac{hc}{\lambda_1} = Wo + (K.E)_1 = Wo + \frac{1}{2}\frac{P^2}{m}$$

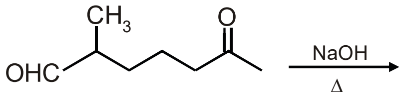
$$\frac{hc}{\lambda_1} = Wo + (K.E)_2 = Wo + \frac{1}{2}\frac{(1.5P)^2}{m}$$

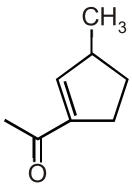
On solving

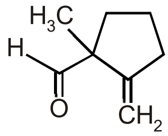
$$\lambda_2 = \frac{4}{9}\lambda_1$$

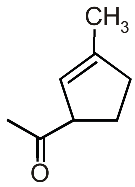
Since the K.E is very high is comparison to work function, then we can assume that K.E + Wo = K.E

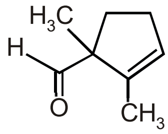
15. The major product obtained in the following reaction is :

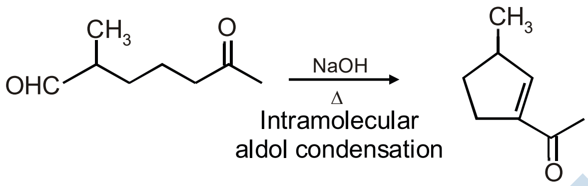


(1*) 

(2) 

(3) 

(4) 

Sol. 

16. The structure of Nylon–6 is

(1) $\left[(CH_2)_6 - \overset{O}{\overset{\|}{C}} - \overset{H}{\overset{|}{N}} \right]_n$

(2) $\left[\overset{O}{\overset{\|}{C}} - (CH_2)_6 - \overset{H}{\overset{|}{N}} \right]_n$

(3) $\left[(CH_2)_4 - \overset{O}{\overset{\|}{C}} - \overset{H}{\overset{|}{N}} \right]_n$

(4*) $\left[\overset{O}{\overset{\|}{C}} - (CH_2)_5 - \overset{H}{\overset{|}{N}} \right]_n$

17. The strength of 11.2 volume solution of H_2O_2 is : [Given that molar mass of H = 1g mol^{-1} and O =16 g mol^{-1}]

(1) 13.6% (2*) 3.4 % (3) 1.7 % (4) 34%

Sol. $\underset{68\ g/L}{2H_2O_2} \longrightarrow 2H_2O + \underset{22.4\ of\ O_2(STP)}{O_2 \uparrow}$

11.2 L of O_2 is given by 34 g of H_2O_2 present in 1 L of H_2O_2 solution.

So % strength of $H_2O_2 = \frac{34}{1000} \times 100 = 3.4g / 100mL$ 3.4%

18. The Mond process is used for the :

(1) Purification of Zr and Ti (2) Extraction of Mo

(3) Extraction of Zn (4*) Purification of Ni

Sol. The mond's process is used for refining of nickel as per following reaction.

$$\underset{\text{Impure}}{Ni} + CO(g) \xrightarrow[330-350K]{\Delta} Ni(CO)_4(\ell) \xrightarrow[450-470]{\Delta} \underset{\text{Pure}}{Ni(s)} + 4CO(g)$$

19. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 k . If C_V = 28 JK^{-1} mol^{-1}, calculate ΔU and ΔpV for this process. (R = 8.0 $JK^{-1}mol^{-1}$)

(1) ΔU = 2.8 kJ : Δ(PV) = 0.8 kJ
(2*) ΔU = 14 kJ : Δ(PV) = 4 kJ
(3) ΔU = 14 J : Δ(PV) = 0.8 J
(4) ΔU = 14 kJ : Δ(PV) = 18 kJ

Sol. $\Delta U = nC_vm \times \Delta T$

$= 5 \times 28 \times 100 = 14$ kJ

$\Delta PV = nR\Delta T$

$= 5 \times 8 \times 100 = 4$ kJ

20. The percentage composition of carbon by mole in methane is :

(1) 25 %
(2*) 20 %
(3) 80 %
(4) 75 %

Sol. In CH_4

Mole of carbon $n_C = 1$

Mole of hydrogen = $n_H = 4$

$$\% \text{ of } n_C = \frac{n_C}{n_C + n_H} \times 100 = \frac{1}{5} \times 100 = 20\%$$

21. The ion that has sp^3d^2 hybridization for the central atom, is :

(1) $[ICl_2]^-$
(2*) $[ICl_4]^-$
(3) $[IF_6]^-$
(4) $[BrF_2]^-$

Sol. $[ICl_4]^- \rightarrow sp^3d^2$

$[IF_6]^- \rightarrow sp^3d^2$

$[ICl_2]^- \rightarrow sp^3d$

$[BrF_2]^- \rightarrow sp^3d$

22. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by :

(1) $k_1k_2[A]$
(2*) $(k_1 + k_2)$ [A]
(3) $(k_1 - k_2)$ [A]
(4) $\left(\frac{k_1}{k_2}\right)$[A]

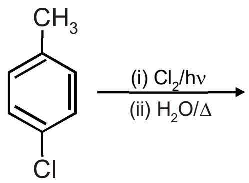
Sol. Applying steady state of approximation

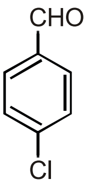
$$\frac{d}{dt}[B] = K_1[A] - K_2[B]$$

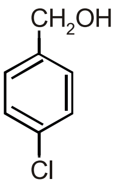
$$O = K_1[A] - K_2[B]$$

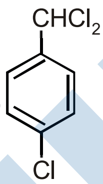
$$\frac{K_1}{K_2}[A] = [B]$$

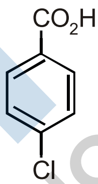
23. The major product of the following reaction is :



(1*) 

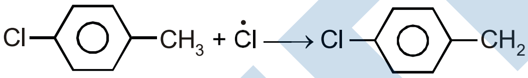
(2) 

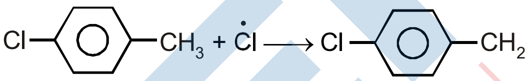
(3) 

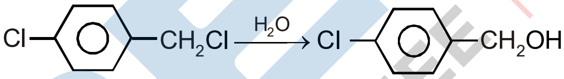
(4) 

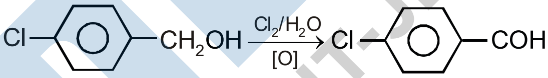
Sol. Reaction involves free radical chlorination followed by hydrolysis.

(a) $Cl_2 \xrightarrow{h\nu} \dot{Cl} + \dot{Cl}$

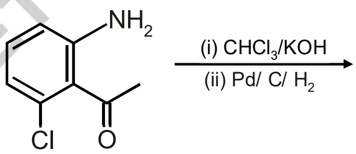
(b) 

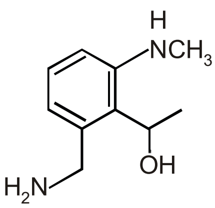
(c) 

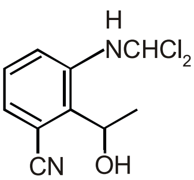
(d) 

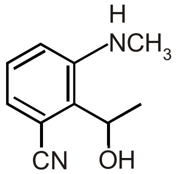
(e) 

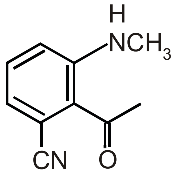
24. The major product obtained in the following reaction is :



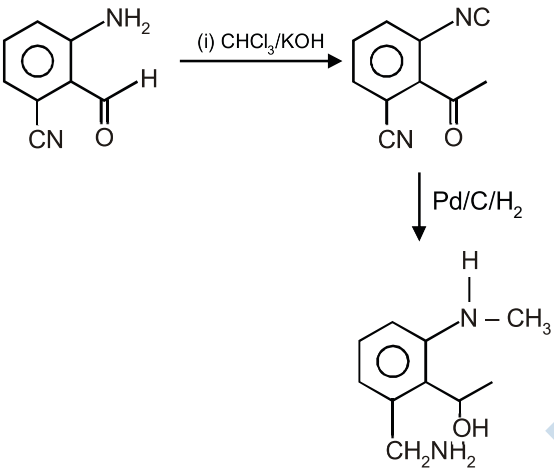
(1*) 

(2) 

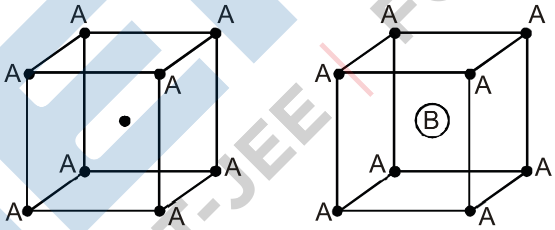
(3) 

(4) 

Sol. The reaction involves carbylamine reaction of 1° amine with $CHCl_3$/KOH followed by reduction with $Pd/C/H_2$.



25. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?



Solid 1 Solid 2

(1) 75% (2*) 90% (3) 45% (4) 65%

Sol. $\%\text{ packing efficiency} = \dfrac{\text{Vol. occupied by atom}}{\text{Vol. of unit cell}} \times 100 \dfrac{\frac{4}{3}\pi r^3}{a^3} \times 100$

Let radius fo corner atom is r and radius of central atom is 2r

So, $\sqrt{3}a = 2(2r) + 2r = 6r$

$a = \dfrac{6r}{\sqrt{3}} = 2\sqrt{3}r$

Now

$$\% \text{ P.E.} = \frac{\frac{4}{3}\pi r^3 + \frac{4}{3}\pi(2r)^3}{\left(2\sqrt{3}r\right)^3} \times 100$$

$$= \frac{\frac{4}{3}\pi\left(r^3 + 8r^3\right)}{8 \times 3\sqrt{3}r^3} \times 100$$

$$= \frac{4\pi \times 9r^3}{3 \times 8 \times 3\sqrt{3}r^3} \times 100 = 90.6\% \approx 90\%$$

26. The compound that inhibits the growth of tumors is:

(1) trans–$[Pd(Cl)_2(NH_3)_2]$ (2*) cis–$[Pt(Cl)_2(NH_3)_2]$

(3) cis–$[Pd(Cl)_2(NH_3)_2]$ (4) trans–$[Pt(Cl)_2(NH_3)_2]$

Sol. cis–$[PtCl_2(NH_3)_2]$ is used in chemotherapy to inhibits the growth of tumors

27. 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

[Density of fatty acid = 0.9 g cm^{-3}, π = 3]

(1) 10^{-8} m (2) 10^{-4} m (3) 10^{-2} m (4*) 10^{-6} m

Sol. Mass of fatty acid = 0.027 g

Radius of plate = 10 cm

Density of fatty acid = 0.9 g/cm^3

Volume of fatty acid $\frac{0.027}{0.9} = 0.03 cm^3$

Area of plate = $\pi r^2 = 3 \times 10^2 = 300\ cm^2$

Height of fatty acid = $\frac{\text{Volume}}{\text{Area}} = \frac{0.03}{300} = 10^{-4} cm = 10^{-6} m$

28. Fructose and glucose can be distinguished by :

(1) Benedict's test (2) Fehling's test (3) Barfoed's test (4*) Seliwanoff's test

Sol. Barfoed's test → Detecting presence of monosaccharides

Fehling's test → For aldehdyes

Benedict's test → For reducing sugars

Seliwanoff's test → Differentiate between aldose and ketose

29. For the following reactions, equilibrium constants are given:

$S(s) + O_2(g) \rightleftharpoons SO_2(g)$; $K_1 = 10^{52}$

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g)$; $K_2 = 10^{129}$

The equilibrium constant for the reaction, $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is

(1) 10^{181} (2*) 10^{25} (3) 10^{77} (4) 10^{154}

Sol. $2SO_2(g) + O_2(g) \longrightarrow 2SO_3(g)$

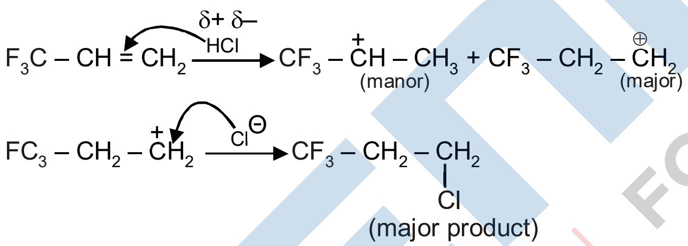
$$K_{eq} = \frac{[SO_3]^2}{[O_2][SO_2]^2}$$

$$= \frac{K_2}{K_1} = \frac{10^{129}}{10^{104}} = 10^{25}$$

30. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product?

(1*) $F_3C–CH = CH_2$ (2) $CH_3O–CH=CH_2$ (3) $H_2N–CH=CH_2$ (4) $Cl–CH=CH_2$

Sol. In this case antimarkonikov product will be formed as major product because carbocation formed at a double bonded carbon having lesser number of H atom will be unstable due to presence of an electron withdrawing group (CF_3) attached to it.



PART-B : MATHEMATICS

31. Given that the slope of the tangent to a curve y = y(x) at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is

(1) $x^2 \log_e |y| = -2(x-1)$
(2) $x \log_e |y| = -2(x-1)$
(3*) $x \log_e |y| = 2(x-1)$
(4) $x \log_e |y| = x - 1$

Sol. $\frac{dy}{dx} = \frac{2y}{x^2}$

$\Rightarrow \ell ny = -\frac{2}{x} + \ell nC$

Passes through (1, 1)

$0 = -2 + \ell nC$

$\Rightarrow \ell ny = \frac{-2}{x} + 2$

$x\ell n |y| = 2(x-1)$

32. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d, e, f are in A.P.
(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
(3) d, e, f are in G.P.
(4*) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Sol. $b^2 = ac$

Also roots of $ax^2 + 2bx + c = 0$ are equal

$\Rightarrow x = \frac{-b}{a}$, common root

$\Rightarrow d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + \int = 0$

$db^2 - 2aeb + fa^2 = 0$, b2 □ ac

$\Rightarrow dac - 2eab + fa^2 = 0$

$\Rightarrow dc - 7eb + fa = 0$

Dividing by ac

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2, \frac{e}{b}$

33. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is

(1*) $4x - 3y - 4 = 0$ (2) $3x - 4y - 1 = 0$ (3) $4x - 3y - 1 = 0$ (4) $3x - 4y - 4 = 0$

Sol. For infinitly many solution

$$\begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = 0$$

$$\Rightarrow K = \frac{-1}{2}$$

Also consider

$x - 2y + k = 1$ and $2x + y + z = 2$

$\Rightarrow 2x - 4y - z - 2$

$2x + y + z = 2$

$\Rightarrow 4x - 3y = 4$

34. Which one of the following statements is not a tautology?

(1*) $(p \vee q) \rightarrow (p \vee (\sim q))$ (2) $(p \wedge q) \rightarrow p$

(3) $p \rightarrow (p \vee q)$ 4) $(p \wedge q) \rightarrow (\sim p) \vee q$

Sol. (A) $(p \vee q) \rightarrow (p \vee (\sim q))$

$= \sim (p \vee q) \vee (p \vee \sim q)$

$= (\sim p \wedge \sim q) \vee (p \vee \sim q)$

$\neq T$

(B) $(p \wedge q) \rightarrow p$

$= \sim (p \wedge q) \vee p = (\sim p \vee \sim q) \vee p$

$= (\sim p \vee p) \vee \sim q$

$= T$

(C) $\sim p \vee (p \vee q)$

$= (\sim p \vee p) \vee q = T$

(D) $\sim (p \vee q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee T = T$

35. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is

(1) $3x - 2y = 0$ (2) $2x - 3y + 10 = 0$ (3) $x - 2y + 8 = 0$ (4*) $2x - y - 2 = 0$

Sol. Let equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

passes through (4, 6)

$\Rightarrow \frac{16}{a^2} - \frac{36}{b^2} = 1$ (i)

Also $e^2 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = 3a^2$ (ii)

From (i) and (ii)

$a^2 = 4, b^2 = 12$

Equation $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Tangent at (4, 6) is x y 1

Or

$2x - y = 2$

36. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is

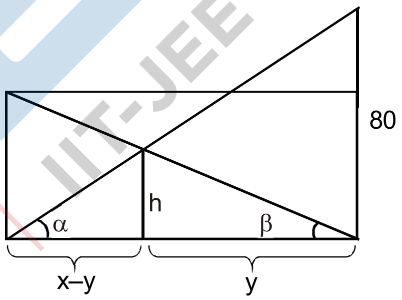
(1) 12 (2*) 16 (3) 15 (4) 18

Sol. $\frac{h}{y} = \frac{20}{y}, \frac{h}{x-y} = \frac{80}{x}$

$\frac{h}{20} = \frac{y}{x}, \frac{h}{80} = \frac{x-y}{x}$

$\frac{h}{20} + \frac{h}{80} = 1$

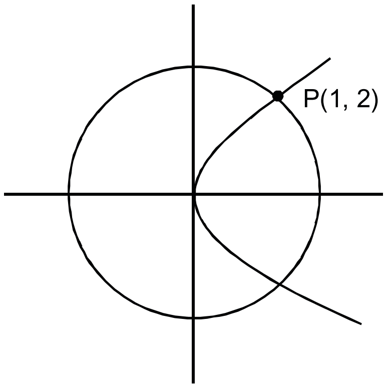
$h\left(\frac{100}{1600}\right) = 1$



37. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point

(1) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (2*) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (3) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Sol.



$x^2 + 4x = 5$

$\Rightarrow x = -5, x = 1$

$\Rightarrow P\ (1, 2)$

Tangent at P is $y = x + 1$

$\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on this.

38. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is

(1*) 4 (2) 3 (3) 2 (4) 5

Sol. $1 - \frac{1}{2^n} > \frac{9}{10}$

$\Rightarrow \frac{1}{10} . \frac{1}{2^n}$

$\Rightarrow 2^n > 10$

$\Rightarrow n = 4$

39. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(1*) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{3}$ (3) $\frac{100}{\sqrt{3}}$ (4) $\frac{10}{3}$

Sol. $AM = \frac{41 + 45 + 54 + 57 + 43 + x}{6} = 48$

$\Rightarrow x = 48$

$\sigma^2 + 48^2 = \frac{1}{6}\left(41^2 + 45^2 + 54^2 + 57^2 + 43^2 + 48^2\right)$

$\sigma^2 = \frac{14024}{6} - 2304 = \frac{100}{3}$

40. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

where C is a constant of integration, then the function f (x) is equal to

(1) $\frac{3}{x^2}$ (2) $-\frac{1}{2x^2}$ (3*) $-\frac{1}{2x^3}$ (4) $-\frac{1}{6x^3}$

Sol. $I = \int \frac{dx}{x^3\left(1+x^6\right)^{2/3}}$

$= \int \frac{dx}{x^7\left(1+\frac{1}{x^6}\right)^{2/3}}$

Put $1+x^{-6} = t \Rightarrow \frac{dx}{x^7} = \frac{-dt}{6}$

$I = \frac{1}{6}\int \frac{-dt}{t^{2/3}} = \frac{-1}{2}\left[1+\frac{1}{x^6}\right]^{1/3} + C$

$= \frac{-1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^2} = xf\left(x\right)\left(1+x^6\right)^{1/3} + C$

$\Rightarrow f\left(x\right) = \frac{-1}{2x^3}$

41. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is

(1) 5 : 9 : 13 (2) 5 : 6 : 7 (3*) 4 : 5 : 6 (4) 3 : 4 : 5

Sol. Given 2b = a + c

Let A = θ, B = $\pi - 3\theta$, C = 2θ

2sinB = sinA + sinC

2sin3θ = sinA + sin2θ

2(3 – 4 $\sin^2\theta$) = (1+ 2cosθ)

$\Rightarrow$ 8cos2 θ – 2cosθ – 3 = 0

$\Rightarrow \cos\theta = \frac{3}{4}$

sinA :sinB :sinC

$\Rightarrow$1: 4 – 3sin2 θ :2cosθ

$\Rightarrow 1 : \frac{5}{4} : \frac{6}{4}$

$\Rightarrow$ 4 : 5 : 6

42. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}}+x^{1/12}\right)^6$ is equal to 200 and x > 1, then the value of x is

(1) 10^4 2) 100 (3) 10^3 (4*) 10

Sol. $^6C_3X-\frac{3}{2}(1+\log x).x^{1/4}=200$

$X^{\frac{1}{4}-\frac{3}{2}(1+\log x)}=10$

$\Rightarrow \frac{1}{4}-\frac{3}{2}(1+\log_{10} x).\log_{10} x=1$

$\Rightarrow 6t^2+5t+4=0,\ t=\log_{10} x$

D < 0

So no real solution

All options are incorrect

43. Let f (x) = $\int_0^x g(t)dt$, where g is a non-zero even function. If f (x + 5) = g(x), then $\int_0^x f(t)dt$ equals

(1) $2\int_5^{x+5} g(t)dt$ (2) $5\int_{x+5}^{5} g(t)dt$ (3*) $\int_{x+5}^{5} g(t)dt$ (4) $\int_5^{x+5} g(t)dt$

Sol. Since g(x) is even with f (0) = 0

f (x) is odd function

g(x) = f (x+ 5)

g(–x) = f(–x + 5)

g(x)= – f (x + 5)

Replace x by x + 5

$\Rightarrow$ f (x) = – g(x + 5)

$\int_0^x f(t)dt=-\int_0^x g(t+5)dt$

$=-\int_5^{x-5} g(t)dt$

$=\int_{x+5}^{5} g(t)dt$

44. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if

(1) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (2) $0<r\le\sqrt{\frac{3}{2}}$ (3*) $r\ge 5\sqrt{\frac{3}{2}}$ (4) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$

Sol. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2+x)\hat{i} - (3-x)\hat{j} - 5\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{2(x^2 - x + 19)}$

$= \sqrt{2}\sqrt{(x-1/2)^2 + 19^{-1/4}} \geq \frac{5\sqrt{3}}{\sqrt{2}}$

45. The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0 is

(1) $\vec{r}\cdot(\hat{i}-\hat{k}) - 2 = 0$ (2) $\vec{r}\times(\hat{i}-\hat{k}) + 2 = 0$ (3*) $\vec{r}\cdot(\hat{i}-\hat{k}) + 2 = 0$ (4) $\vec{r}\times(\hat{i}+\hat{k}) + 2 = 0$

Sol. P_1:x + y + z = 1

P_2 :2x + 3y + 4z = 5

Required plane is 1 2 $P_1 + \lambda P_2 = 0$

$\Rightarrow$(1+ 2λ) x + (1+ 3λ) y + (1+ 4λ)z = 1+ 5λ .....(i)

which is perpendicular to x – y + z = 0

$\Rightarrow$1+ 2λ – (1+ 3λ) + 1+ 4λ = 0

$\Rightarrow \lambda = \frac{-1}{3}$

(i) $\Rightarrow x - z + 2 = 0$

$\vec{r}.(\hat{i}-\hat{k}) + 2 = 0$

46. Let f : R → R be a differentiable function satisfying f '(3) + f '(2) = 0. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

(1) e (2) e^{-1} (3*) 1 (4) e^2

Sol. 1^∞ Form

$$k = \lim_{x\to\infty}\left(\frac{f(3+x)-f(2x)-f(3)(f(2))}{x(1+f(2-x)-f(2))}\right)$$

$$= \lim_{x\to\infty}\frac{f'(3+x)+f'(2-x)}{(1+f(2-x)-f(2))-xf'(2-x)} = 0$$

$\Rightarrow e^k = 1$

47. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then $(1 + iz + z^5 + iz^8)^9$ is equal to

(1) $(-1 + 2i)^9$ (2*) -1 (3) 0 (4) 1

Sol. $z = \frac{\sqrt{3}+i}{2} = e^{\frac{i\pi}{6}}$

$(1 + iz + z^5 + iz^8)^9$

$= (1+e^{i\pi/2}\, e^{i\pi/6} + e^{i5\pi/6} + e^{i\pi/2}\, e^{i8\pi/6})^9$

$= \left(1 + e^{i2\pi/6} + e^{i5\pi/6} + e^{i\frac{11\pi}{6}}\right)$

$= \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^9 = \left(E^{I\pi/3}\right)^9 = e^{i3\pi} = -1$

48. Let f : [–1, 3] → R be defined as $f(x) = \begin{cases} |x| + [x], -1 \le x < 1 \\ x + |x|, \ 1 \le x < 2 \\ x + [x], \ 2 \le x \le 3 \end{cases}$,

where [t] denotes the greatest integer less than or equal to t. Then f, is discontinuous at

(1*) only three points (2) only one point (3) only two points (4) four or more points

Sol. $f(x) = \begin{cases} -x-1 & x \in [-1, 0] \\ x & x \in [0, 1] \\ 2x & x \in [1, 2] \\ x+2 & x \in [2, 3] \end{cases}$

f (x) is discontinuous at x = 0, 1

49. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is

(1) 10 (2*) 5 (3) 8 (4) 6

Sol. $be = 5\sqrt{3}$

$b^2e^2 = 75$

$(b - a)(b + a) = 75 \Rightarrow b + a = 15$

$\Rightarrow b = 10, a = 5$

$LR = \frac{2a^2}{B} = 5$

50. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det (A) ∈ [2, 16], then c lies in the interval

(1) $[3, 2 + 2^{3/4}]$ (2) [2, 3) (3*) [4, 6] (4) $(2 + 2^{3/4}, 4)$

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & b-2 & c-2 \\ 4 & b^2-4 & c^2-4 \end{vmatrix}$

$= (b-2)(c-2)\begin{vmatrix} 1 & 1 \\ b+2 & c+2 \end{vmatrix}$

$|A| = (b-2)(c-2)(c-b)$

2,b,c are in AP $\Rightarrow$ 2,2 + d, 2 + 2d

$\Rightarrow |A| = (d)(2d)(d) = 2d^3 \in [2, 16]$

$\Rightarrow d^3 \in [1, 8]$

$\Rightarrow 2d \in [2, 4]$

$\Rightarrow 2 + 2d \in [4, 6]$

51. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then the distance of R from the origin is

(1) 6 (2) $\sqrt{53}$ (3) $2\sqrt{21}$ (4*) $2\sqrt{14}$

Sol. Equation of PQ is $\frac{x-2}{6} = \frac{y+3}{3} = \frac{z-4}{6}$

R (4, y, z) line on this

$\Rightarrow \frac{1}{3} = \frac{y+3}{3} = \frac{z-4}{6}$

$\Rightarrow$ R (4, –2, 6)

$QR = \sqrt{16+4+36} = 2\sqrt{14}$

52. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to

(1*) $2-\frac{11}{2^{19}}$ (2) $2-\frac{21}{2^{20}}$ (3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{3}{2^{17}}$

Sol. $S = \sum_{k=1}^{20} \frac{k}{2^k}$

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots.. + \frac{20}{2^{20}}$

$S\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots.. + \frac{20}{2^{21}}$

$$\frac{1}{2}S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{20}} - \frac{20}{2^{21}}$$

$$\frac{1}{2}S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{20}} - \frac{20}{2^{21}}$$

$$= \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{\frac{1}{2}} - \frac{20}{2^{21}}$$

$$= 1 - \frac{21}{2^{21}}$$

$$s = 2 - \frac{11}{2^{19}}$$

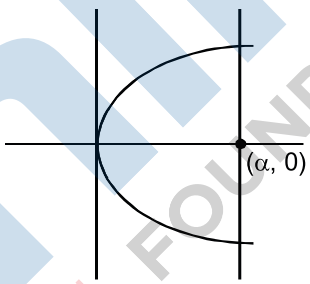
53. Let $S(\alpha) = \{(x, y) : y^2 \le x,\ 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{2}{5}\right)^{1/3}$ (2) $4\left(\frac{2}{5}\right)^{1/3}$ (3*) $4\left(\frac{4}{25}\right)^{1/3}$ (4) $2\left(\frac{4}{25}\right)^{1/3}$

Sol. $S(\lambda) = 2\int_0^{\lambda}\sqrt{x}dx = \frac{4}{3}\lambda^{3/2}$

$\frac{S(\lambda)}{S(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$

$\Rightarrow \lambda = 4\left(\frac{4}{25}\right)^{1/3}$



54. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals

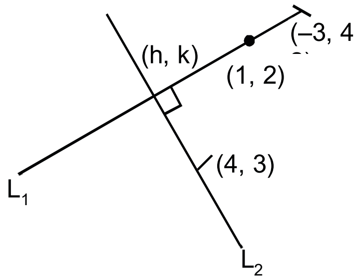
(1) 0 (2) $-\frac{1}{7}$

(3*) 3 (4) $\frac{1}{3}$

Sol. Equation of L_1 is $x + 2y = 5$ and equation of L_2 is $2x - y = 5$

Their point of intersection is (3, 1)

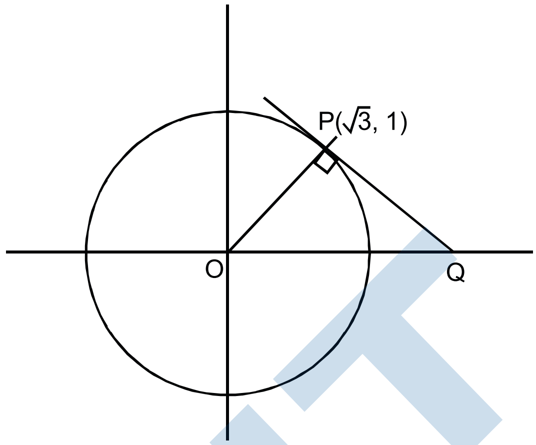
$\Rightarrow \frac{k}{h} = \frac{1}{3}$



55. The tangent and the normal lines at the point $\left(\sqrt{3}, 1\right)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{3}$ (3) $\frac{4}{\sqrt{3}}$ (4*) $\frac{2}{\sqrt{3}}$

Sol. Slope of OP $= \frac{1}{\sqrt{3}}$



Equation of PQ is

$y - 1 = -\sqrt{3}\left(x - \sqrt{3}\right)$

$\Rightarrow y + \sqrt{3}x = 4$

$\Rightarrow Q\left(\frac{4}{\sqrt{3}}, 0\right)$

Area $= \frac{2}{\sqrt{3}}$

56. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is

(1) 360 (2) 288 (3) 306 (4*) 310

Sol. Starting with 5 = 6^3 = 216

Starting with 44 = 6^2 = 36

Starting with 45 = 6^2 = 36

Starting with 43 = 18 (Not using 0, 1, 2)

Starting with 432 = 4

Total = 310

57. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1*) $2f_1(x) f_1(y)$ (2) $2f_1(x + y) f_2(x - y)$ (3) $2f_1(x) f_2(y)$ (4) $2f_1(x + y) f_1(x - y)$

Sol. $f_1(x)\frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

$f_1(x + y) + f_1(x - y)$

$= \frac{1}{2}\left(a^{x+y} + a^{-x-y} + a^{x-y} + a^{-1+y}\right)$

$= \frac{1}{2}\left(a^x(a^y + a^{-y}) + a^{-x}\left(a^y + a^{-y}\right)\right)$

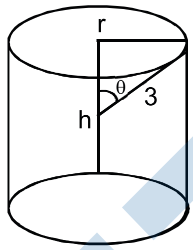
$$= 2.\left(\frac{a^x + a^{-x}}{2}\right)\left(\frac{a^y + a^{-y}}{2}\right)$$

$= 2f_1(x)f_1(y)$

58. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $\sqrt{6}$ (2*) $2\sqrt{3}$ (3) $\frac{2}{3}\sqrt{3}$ (4) $\sqrt{3}$

Sol. h = 2(3cos) = 6cos θ, r = 3 sinθ



$V = \pi r^2 h$

$= \pi\,(4\sin^2\theta)\,(6\cos\theta)$

$= 54\pi\sin^2\theta\cos\theta$

$$\frac{dv}{d\theta} = 0 \Rightarrow 2\sin\theta\cos\theta - \sin^3\theta = 0$$

$\Rightarrow 2\sin\theta - 3\sin^3\theta = 0$

$$\Rightarrow \sin\theta \pm \sqrt{\frac{2}{3}}$$

$$\cos\theta = \sqrt{\frac{1}{3}}$$

$$h = \frac{6}{\sqrt{3}}$$

59. The number of integral values of m for which the equation $(1 + m^2)\,x^2 - 2(1 + 3m)\,x + (1 + 8m) = 0$ has no real root is

(1) 2 (2*) infinitely many (3) 1 (4) 3

Sol. $D = 4\,(1 + 3m)^2 - 4(1 + m^2)\,(1 + 8m)$

$= 4(1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3)$

$= -8m\,(4m^2 - 4m + 1)$

$= -8m(2m - 1)^2 < 0$

$\therefore$ Infinitely many values of m.

60. If f(1) = 1, f '(1) = 3, then the derivative of $f\big(f(f(x))\big) + \big(f(x)\big)^2$ at x = 1 is

(1*) 33 (2) 9 (3) 12 (4) 15

Sol. $f = f(f(f(x))) + (f(x))^2$

$$\frac{dy}{dx} = f'\big(f(f(x))\big)f'\big(f(x)\big)f'(x) + 2f(x)f'(x)$$

Put x = 1

$$\frac{dy}{dx} = 27 + 6 = 33$$

PART-C : PHYSICS

61. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth is closest to :

[Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/k Avogadro Number $N_A = 6.02 \times 10^{26}$ / Kg Radius of Earth; 6.4×10^6 m Gravitational acceleration on Earth = 10 ms^{-2}]

(1) 3×10^5 K (2) 800 K (3) 650 K (4*) 10^4 K

Sol. $v_{rms} = \sqrt{\frac{3RT}{m}}$ $\quad v_{escaps} = \sqrt{2gR_e}$

$v_{rms} = v_{escape}$

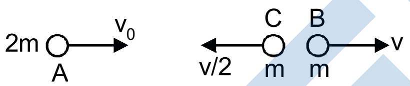
$$\frac{3RT}{m} = 2gR_e$$

$$\frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}}{2} \times 10 \times 10^3 = 10^4 k$$

62. A nucleus A, with a finite de–Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de–Broglie wavelengths λ_B and λ_C of B and C are respectively

(1) $\lambda_A, \frac{\lambda_A}{2}$ (2) λ_A, $2\lambda_A$ (3) $2\lambda_A$, λ_A (4*) $\frac{\lambda_A}{2}, \lambda_A$

Sol.



Let mass of B and C is m each. By momentum conservation

$$2mv_0 = mv - \frac{mv}{2}$$

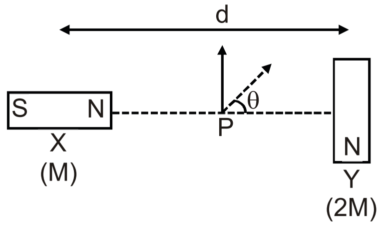
$v = 4v_0$

$P_A = 2mv_0$ $p_B = 4mv_0$ $p_c = 2mv_0$

De-Broglie wavelength $\lambda = \frac{h}{p}$

$$\lambda_A = \frac{h}{2mv_0}; \lambda_B = \frac{h}{4mv_0}; \lambda_C = \frac{h}{2mv_0}$$

63. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle $\theta = 45^0$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)



(1) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(2) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

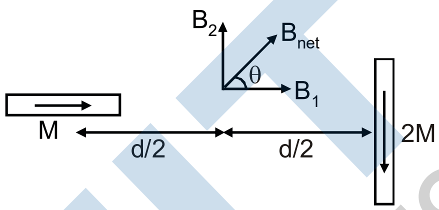
(3*) 0

(4) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

Sol. $B_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}; B_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}$

$B_1 = B_2$

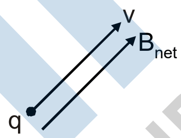
$\Rightarrow B_{net}$ is at 45° (θ = 45°)



Velocity of charge and B_{net} are parallel so

by

$\vec{F} = q(\vec{b}\times\vec{B})$ force on charge particle is zero.



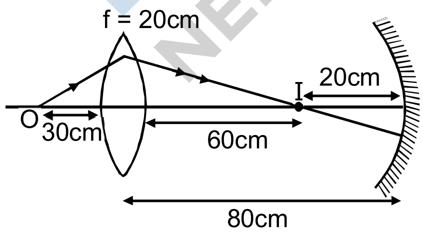
64. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :

(1*) 10 cm (2) 25 cm (3) 20 cm (4) 30 cm

Sol. Image formed by lens

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}; \frac{1}{v}+\frac{1}{30}=\frac{1}{20}$

v = + 60 cm

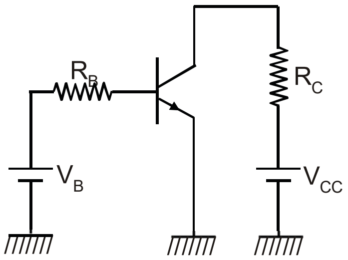


If image position does not change even when mirror is removed it means image formed by lens is formed at centre of curvature of spherical mirror. Radius of curvature of mirror = 80 – 60 = 20 cm.

$\Rightarrow$ Focal length of mirror f = 10 cm for virtual image, object is to be kept between focus and pole.

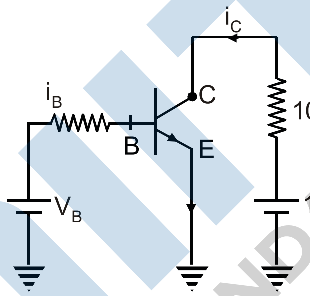
$\Rightarrow$ Maximum distance of object from spherical mirror for which virtual image is formed, is 10 cm.

65. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, R_C= 1 kΩ and V_{CC} = 10V. What is the minimum base current for V_{CE} to reach saturation?



(1) 7 μA (2) 100 μA (3*) 40 μA (4) 10 μA

Sol. At saturation state, V_{CE} becomes zero



$$\Rightarrow i_C = \frac{10V}{1000\Omega} = 10 \text{ mA}$$

Now current gain factor $\beta = \frac{i_C}{i_B}$

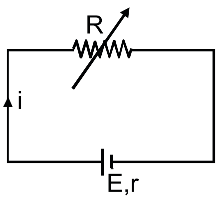
$$\Rightarrow i_B = \frac{10 \text{ mA}}{250} = 40 \text{ } \mu A$$

66. A cell of internal resistance r drives current through an external resistance R . The power delivered by the cell to the external resistance will be maximum when :

(1*) R = r (2) R = 0.001 r (3) R = 2r (4) R = 1000 r

Sol. Current $i = \frac{E}{r+R}$

Power generated in R



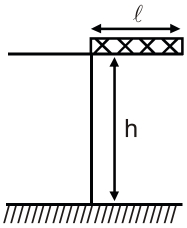
$$P = \frac{E^2R}{(r+R)^2}$$

For maximum power $\frac{dp}{dR} = 0$

$$E^2\left[\frac{(r+R)^2 \times 1 - R \times 2(r+R)}{(r+R)^4}\right] = 0$$

$\Rightarrow$ r = R

67. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slip off the table in a very short time τ = 0.01 s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :



(1) 0.28 (2) 0.02 (3*) 0.5 (4) 0.3

Sol. Angular impulse = change in angular momentum.

$\tau \Delta t = \Delta L$

$$mg\frac{\ell}{2}\times 0.01 = \frac{m\ell^2}{3}\omega$$

$$\omega = \frac{3g\times 0.01}{2\ell} = \frac{3\times 10\times 0.01}{2\times 0.3} = \frac{1}{2} = 0.5 \text{ red / s}$$

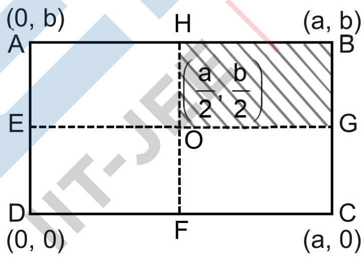
Time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1 \text{ sec.}$$

In this time angle rotate by rod

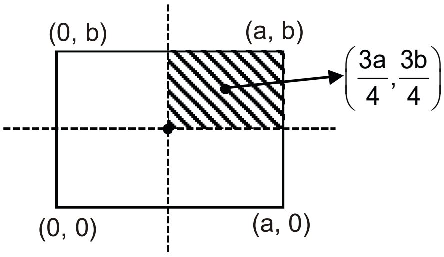
$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

68. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut–off, the coordinates of the centre of mass of the remaining portion will be :



(1) $\left(\frac{5a}{3},\frac{5b}{3}\right)$ (2) $\left(\frac{2a}{3},\frac{2b}{3}\right)$ (3) $\left(\frac{3a}{4},\frac{3b}{4}\right)$ (4*) $\left(\frac{5a}{12},\frac{5b}{12}\right)$

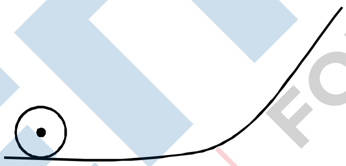
Sol. $$x = \frac{M\frac{a}{2} - \frac{M}{4}\times\frac{3a}{4}}{m - \frac{M}{4}}$$



$$= \frac{\frac{a}{2} - \frac{3a}{16} \times \frac{3a}{4}}{\frac{3}{4}} = \frac{5b}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

69. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by :



(1) 1   2) $\frac{2}{\sqrt{5}}$ (3) $\frac{4}{5}$ (4*) $\frac{14}{15}$

Sol. For solid sphere

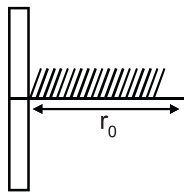
$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph}$$

For solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{cyl}$$

$$\Rightarrow \frac{h_{sph}}{h_{cyl}} = \frac{7/5}{3/2} = \frac{14}{15}$$

70. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :



(1) $v \propto \left(\frac{r}{r_0}\right)$ (2) $v \propto e^{+r/r_0}$ (3*) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$ (4) $v \propto \ell n\left(\frac{r}{r_0}\right)$

Sol. $\frac{1}{2}mV^2 = -q(V_f - V_i)$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$\Delta v = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$

$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$

$c \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

71. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be : (Radius of the Earth = 6.4×10^6 m)

(1) 40 m (2*) 32 m (3) 51 m (4) 20 m

Sol. Range $= \sqrt{2Rh_T} + \sqrt{2Rh_R}$

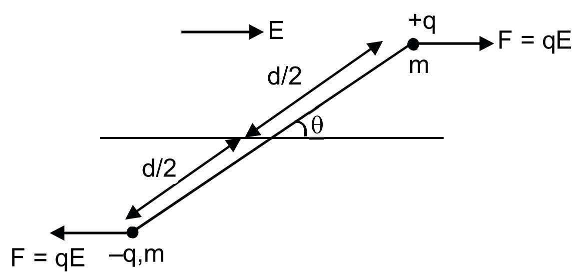
$50 \times 10^3 = \sqrt{2 \times 6400 \times 10^3 \times h_T} + \sqrt{2 \times 6400 \times 10^3 \times 70}$

By solving $h_T = 32$ m.

72. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E If it is slightly rotated from its equilibrium orientation then its angular frequency is ω:

(1) $2\sqrt{\frac{qE}{md}}$ (2*) $\sqrt{\frac{2qE}{md}}$ (3) $\sqrt{\frac{qE}{2md}}$ (4) $\sqrt{\frac{qE}{md}}$

Sol. Moment of inertia



$(\ell) = m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by $\tau = \ell\alpha$

$(qE)(d \sin\theta) = \frac{md^2}{2}.\alpha$

$\alpha = \left(\frac{2qE}{md}\right)\sin\theta$ for small θ

$\Rightarrow \alpha\left(\frac{2qE}{md}\right)\theta$

$\Rightarrow$ Angular frequency $\omega = \frac{\sqrt{2qE}}{md}$

73. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :

(1) 50 s (2) 10 s (3*) 20 s (4) 100 s

Sol. A = A0$e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds]

$\frac{A_0}{2} = A_0 e^{-\gamma(2)}$; $2 = e^{2\gamma}$

$\ell n2 = 2\gamma$; $\gamma = \frac{\ell n2}{2}$

$\because A = A_0 e^{-\gamma t}$

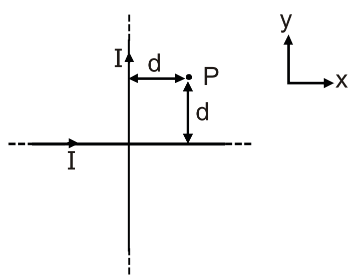
$\ell n\frac{A_0}{A} = \gamma t$; $\ell n1000 \frac{\ell n}{2} t$

$2\left(\frac{3\ell n10}{\ell n2}\right) = t$; $\frac{6\ell n10}{\ell n2} = t$

t = 19.931 sec

$t \approx 20$ sec

74. Two very long, straight, and insulated wires are kept at 90^0 angle from each other in xy–plane as shown in the figure.

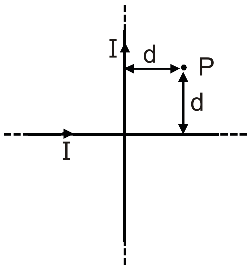


These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be

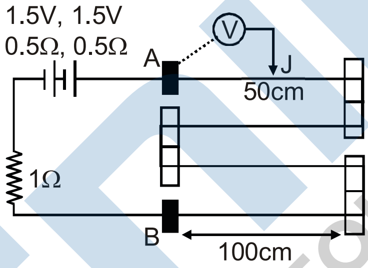
(1) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ (2) $\frac{+\mu_0 I}{\pi d}(\hat{z})$ (3*) Zero   (4) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

Sol. Magnetic field at point P

$$\vec{B}_{net} = \frac{\mu_0 i}{2\pi d}(-\hat{k}) + \frac{\mu_0 i}{2\pi d}(\hat{k}) = 0$$



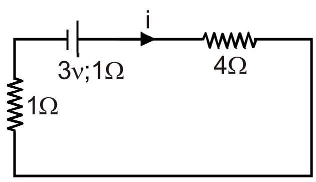
75. In the circuit shown, a four–wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01Ω /cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be:



(1) 0.50 V   (2) 0.20 V   (3) 0.75 V   (4*) 0.25 V

Sol. Resistance of wire AB = 400 × 0.01 = 4Ω

$i = \frac{3}{6} = 0.5\,A$



Now voltmeter reading = I (Resistance of 50 cm lenght)

= (0.5 A) (0.01 × 50) = 0.25 volt

76. A circuit connected to an ac source of emf e = $e_0$sin (100t) with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?

(1) RL circuit with R = 1 kΩ and L = 1 mH   (2*) RC circuit with R = 1 kΩ and C = 10 μF

(3) RL circuit with R = 1 kΩ and L = 10 mH   (4) RC circuit with R = 1 kΩ and C = 1 μF

Sol. Given phase difference $= \frac{\pi}{4}$ and ω = 100 rad/S

⇒ Reactance (X) = Resistance (R)

Now by checking option.

Option (A)

$R = 10^3\Omega$ and $X_c = \dfrac{1}{10^{-6}\times 100} = 10^4\Omega$

Option (B)

$R = 10^3\ \Omega$ and $X_L = 10 \times 10^{-3} \times 100 = 1\Omega$

Option (D)

$R = 10^3\ \Omega$ and $X_C = \dfrac{1}{10\times 10^{-6}\times 100} = 10^3\Omega$

77. If Surface tension (S), Moment of Inertia (I) and planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :

(1) $S^{3/2}I^{1/2}h^0$ (2*) $S^{1/2}I^{1/2}h^0$ (3) $S^{1/2}I^{1/2}h^{-1}$ (4) $S^{1/2}I^{3/2}h^{-1}$

Sol. $p = k\, s^a I^b h^c$

Where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a(ML^2)^b(ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \dfrac{1}{2}\quad b = \dfrac{1}{2}\quad c = 0$

$S^{1/2}I^{1/2}h^0$

78. The ratio of mass densities of nuclei of $^{40}C_a$ and ^{16}O is close to :

(1*) 1 (2) 2 (3) 5 (4) 0.1

Sol. Mass densities of all nuclei are same so their ratio is 1.

79. Let $|\vec{A}_1|=3, |\vec{A}_2|=5$ and $|\vec{A}_1+\vec{A}_2|=5$. The value of $\left(2\vec{A}_1+3\vec{A}_2\right)\cdot\left(3\vec{A}_1-2\vec{A}_2\right)$ is :

(1) –106.5 (2) –112.5 (3) –99.5 (4*) –118.5

Sol. $\left|\vec{A}_1\right| = 3, \left|\vec{A}_2\right| = 5$, and $\left|\vec{A}_1+\vec{A}_2\right| = 5.1$

$\left|\vec{A}_1+\vec{A}_2\right| = \left|\vec{A}_1\right|^2 + \left|\vec{A}_2\right|^2 + 2\left|\vec{A}_1\right|\left|\vec{A}_2\right|\cos\theta$

$\cos\theta = -\dfrac{3}{10}$

$\left(2\vec{A}_1+3\vec{A}_2\right)\cdot\left(3\vec{A}_1-2\vec{A}_2\right)$

$= 6|\vec{A}_1|^2 + 9\vec{A}_1 \cdot \vec{A}_2 - 4\vec{A}_2 - 6|\vec{A}_2|^2$

$= -118.5$

80. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5v_1$, then v_1 is :

(1*) $v_4 - v_2$ (2) $v_4 + v_2$ (3) $v_4 - \frac{v_2}{2}$ (4) $v_4 - \frac{v_2}{4}$

Sol. Applying linear momentum conservation

$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} = m_2v_4\hat{i}$

$m_1v_1 + 0.5\ m_1v_2 = m_1(0.5\ v_1) + 0.5\ m_1v_4$

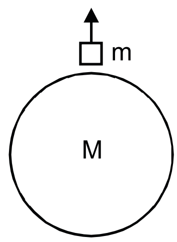
$0.5\ m_1v_1 = 0.5\ m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

81. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(1*) $\frac{E}{16}$ (2) $\frac{E}{32}$ (3) $\frac{E}{64}$ (4) $\frac{E}{4}$

Sol. Minimum energy required (E) = – (Potential energy of object at surface of earth)



$= -\left(-\frac{GMm}{R}\right) = \frac{GMn}{R}$

Now $M_{earth} = 64\ M_{moon}$

$\rho \cdot \frac{4}{3}\pi R_e^3 = 64 \cdot \frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$

Now $\frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{E_{earth}} \cdot \frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$

$\Rightarrow \quad E_{moon} = \frac{E}{16}$

82. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to :

(1*) 6.8 % (2) 3.5 % (3) 0.7 % (4) 0.2 %

Sol. $T = \frac{30\text{ sec}}{20}$ $\Delta T = \frac{1}{20}\text{sec},$

L = 55 cm ΔL = 1 mm = 0.1 cm

$$g = \frac{4\pi^2 L}{T^2}$$

Percentage error in g is

$$\frac{\Delta g}{g}\times 100 = \left(\frac{\Delta L}{L}+\frac{2\Delta T}{T}\right)100\%$$

$$= \left(\frac{0.1}{55}+\frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right)100\% \approx 6.8\%$$

83. Calculate the limit of resolution of a telescope objective having a diameter of a 200cm, if it has to detect light of wavelength 500 nm coming from a star.

(1) 457.5 × 10^{-9} radian (2) 152.5 × 10^{-9} radian

(3*) 305 × 10^{-9} radian (4) 610 × 10^{-9} radian

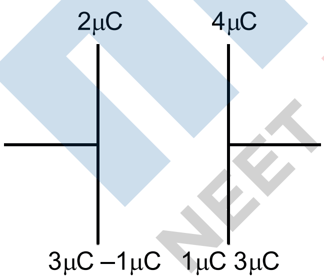
Sol. Limit of resolution of telescope $= \frac{1.22\lambda}{D}$

$$\theta = \frac{1.22\times 500\times 10^{-9}}{200\times 10^{-2}} = 305\times 10^{-9}\text{ radian}$$

84. A parallel plate capacitor has 1μF capacitance. One of its two plates is given + 2μC charge and the other plate, + 4μC charge. The potential difference developed across the capacitor is :

(1) 5 V (2*) 1 V (3) 3 V (4) 2 V

Sol.



Charges at inner plates are 1 μC and –1 μC.

∴ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\mu C}{1\mu F} = \frac{1\times 10^{-6}\,C}{1\times 10^{-6}\text{ Farad}} = 1\,V$$

85. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :

(1*) 1.7 mm (2) 1.3 mm (3) 1.9 mm (4) 1.5 mm

Sol. Given:

$\frac{Y_A}{Y_B} = \frac{7}{4}$ $L_A = 2m$ $A_A = \pi R^2$

$L_B = 1.5\ m$ $A_B = \pi(2mm))^2$

$$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

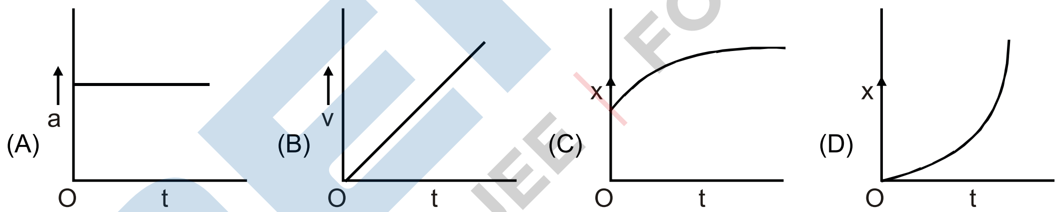
$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$$

$$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{2}Y_B\right)}{2} = \frac{\pi(2\ mm)^2 \cdot Y_B}{1.5}$$

R = 1.74 mm

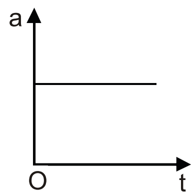
86. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x – axis. Identify all figures that correctly represent the motion qualitatively.

(a = acceleration, v = velocity, x = displacement, t = time)



(1*) (A), (B), (D) (2) (B), (C) (3) (A), (B), (C) (4) (A)

Sol. Given initial velocity u = 0 and acceleration is constant



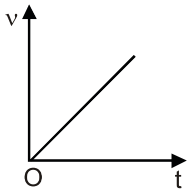
At time t

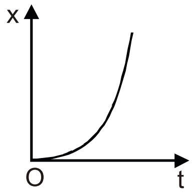
v = 0 + at

$\Rightarrow$ v = at

Also $x = 0(t) + \frac{1}{2}at^2$

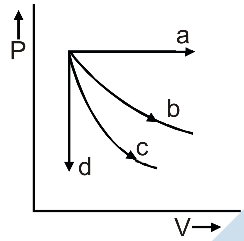
$$\Rightarrow x = \frac{1}{2}at^2$$





Graph (a), (b) and (d) are correct.

87. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :



(1) a d b c (2*) d a b c (3) d a c b (4) a d c b

Sol. isochoric $\rightarrow$ Process d

Isobaric $\rightarrow$ Process a

Adiabatic slope will be more than isothermal so

Isothermal $\rightarrow$ Process b

Adiabatic $\rightarrow$ Process c

Order $\rightarrow$ d a b c

88. The magnetic field of an electromagnetic waves is given by :

$$\vec{B} = 1.6\times10^{-6}\cos(2\times10^{7}z + 6\times10^{15}t)(2\hat{i}+\hat{j})\frac{Wb}{m^2}$$

The associated electric field will be :

(1) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z - 6\times10^{15}t)(2\hat{i}+\hat{j})\frac{V}{m}$

(2*) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z + 6\times10^{15}t)(-\hat{i}+2\hat{j})\frac{V}{m}$

(3) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z + 6\times10^{15}t)(\hat{i}-2\hat{j})\frac{V}{m}$

(4) $\vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z - 6\times10^{15}t)(-2\hat{j}+\hat{i})\frac{V}{m}$

Sol. If we use that direction of light propagation will be alongv $\vec{E}\times\vec{B}$ Then (A) option is correct.

Magnitude of E = CB

$E = 3 \times 10^8 \times 1.6 \times 10^{-6} \times \sqrt{5}$

$E = 4.8 \times 10^2 \sqrt{5}$

$\vec{E}$ and $\vec{B}$ are perpendicular to each other

$\Rightarrow \vec{E} \cdot \vec{B} = 0$

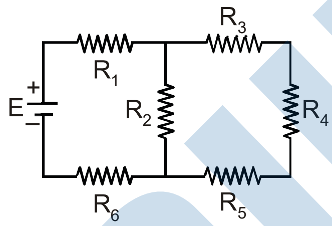
$\Rightarrow$ Either direction of E is $\hat{i} - 2\hat{j}$ or $-\hat{i} + 2\hat{j}$ from given option

Also wave propagation direction is parallel to $\vec{E} \times \vec{B}$ which is $-\hat{k}$

$\Rightarrow \vec{E}$ is along $\left(-\hat{i} + 2\hat{j}\right)$

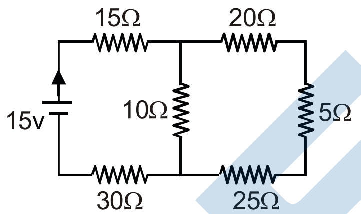
89. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given :

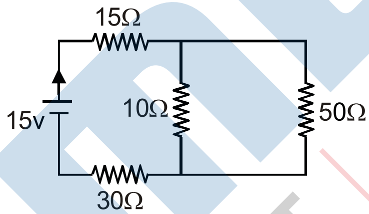
$R_1 = 15\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\Omega$, $R_5 = 25\ \Omega$, $R_6 = 30\ \Omega$, $E = 15$ V

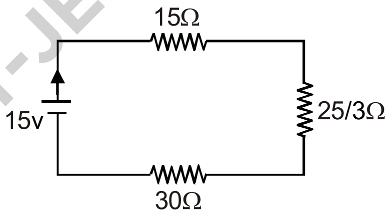


(1) 20 / 3 (2) 13 / 24 (3) 7 / 18 (4*) 9 /32

Sol.







$$R_{eq} = 15 + \frac{25}{3} + 30 = \frac{45 + 25 + 90}{3} = \frac{160}{3}$$

$$I = \frac{E}{R_{eq}} = \frac{15 \times 3}{160} = \frac{9}{32} \text{amp.}$$

90. The electric field in a region is given by $\vec{E} = (Ax + b)\ \hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2 then $V_1 - V_2$ is

(1) –48 V (2) 320 V (3*) 180 V (4) –520V

Sol. $\vec{E} = (20X + 10)\hat{i}$

$$V_1 - V_2 = -\int_{-5}^{1} (20x + 10)\,dx$$

$$V_1 - V_2 = -(10x^2 + 10x)^{1}_{-5}$$

$$V_1 - V_2 = 10(25 - 5 - 1 - 1)$$

$$V_1 - V_2 = 180\text{ V}$$